

المعادلات التفاضلية الجزئية \* Partial Differential equation \*

المعادلات التفاضلية الجزئية P.D.E

order

$$\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial u}{\partial y} = 0$$

2nd order

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^3 u}{\partial y^3} + 2u = 0$$

3rd order

$$\frac{\partial^2 u}{\partial x \partial y} = u_{xy} \Rightarrow \frac{\partial u}{\partial x} = u_x \frac{\partial}{\partial x} = 0$$

\* Degree

$$u_{xx} + 2(u_x)^3 = (u_{xy})^2$$

2nd order , 2nd degree

(1)

Formation p. D. E by elementing  
Constant  
تكوين المعادلات التفاضلية بحذف الثوابت

\* نقاض بعد مرات الدوران

Ex obtain p. D. E if

$u = f(x+2y)$  is solution

أو

نقاض بالـ  $x$  :  $\frac{\partial u}{\partial x} = f'(x+2y)$

نقاض بالـ  $y$  :  $\frac{\partial u}{\partial y} = 2f'(x+2y)$

$\therefore \frac{\partial u}{\partial y} = 2 \frac{\partial u}{\partial x} \quad \#$

Ex  $u = f(x-2y) + g(x+2y)$

أو

p. D. E  $u_x = f'(x-2y) + g'(x+2y)$   
 $u_y = -2f'(x-2y) + 2g'(x+2y)$

©



$$u_{xx} = f''(x-2y) + g''(x+2y)$$

$$u_{yy} = 4 f''(x-2y) + 4g''(x+2y)$$

$$\therefore u_{yy} = 4 u_{xx}$$


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if  $u = y \cdot f(x) \quad \text{①}$

بالتفصيل: لـ ١

$$u_y = f(x)$$

بالتعويض: ①

$$u = y \cdot u_y$$


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\*  $u = e^{2x} f(x-y)$

$$u_x = 2e^{2x} f(x-y) + e^{2x} f'(x-y)$$

$$u_y = -e^{2x} f'(x-y)$$

$$\therefore u_x = 2u - u_y \quad \# \quad \text{②}$$

Solution of P.D.E

حل المعادلة التفاضلية  
P.D.E. حقه

Show that  $u = e^{2x} \cos y$  is  
solution  $u_{xx} + 4u_{yy} = 0$

$$u_x = 2 e^{2x} \cos y$$

$$u_y = e^{2x} \sin y$$

$$u_{xx} = 4 e^{2x} \cos y$$

$$u_{yy} = -e^{2x} \cos y$$

$$\therefore u_{xx} = -4 u_{yy}$$

$$\therefore u_{xx} + 4 u_{yy} = 0$$

$u = e^{2x} \cos y$  حقه المعادلة

$$u_{xx} + 4 u_{yy}$$

الحل

Q.E.D.



classification of P.D.E

تصنيف المعادلات التفاضلية الجزئية

$$a. u_{xx} + 2b u_{xy} + c u_{yy} = 0$$

$$b^2 - ac \Rightarrow \begin{cases} > 0 & \text{hyperbolic} \\ < 0 & \text{elliptic} \\ = 0 & \text{parabolic} \end{cases}$$

\* classify  $u_{xx} + u_{yy} = 0$

معادلة لابلاس

$$a=1, \quad b=1, \quad c=0$$

$$b^2 - ac = 0 - 1 < 0$$

elliptic

$$u_{xx} = \frac{1}{k} u_t \quad (\text{Heat equation})$$

$$u_{xx} - \frac{1}{k} u_t = 0$$

$$a = 1$$

$$b = 0$$

$$h = 0$$

$$b^2 - 4ac = 0 \quad \text{Parabolic}$$

$$\dagger u_{xx} = \frac{1}{c^2} u_{tt} \quad (\text{Wave equation})$$

$$a = 1$$

$$b = -1/c^2$$

$$h = 0$$

$$b^2 - 4ac = 0 + \frac{1}{c^2} > 0 \quad \text{Hyperbolic}$$

$$\dagger 2u_{xx} + 4u_{xy} + bu_{yy} = 0$$

$$a = 2, \quad h = 2, \quad b = b$$

$$b^2 - 4ac = 4 - 2b$$

⑦

$b \geq 2$  parb.

$b > 2$  ellip

$b < 2$  hyperb.



+ classify P.D.E

$$x \cdot u_{xx} + 2y u_{xy} + x u_{yy} = 0$$

$$a = b = x \quad h = y$$

$$h^2 - ab = y^2 - x^2$$

i f  $y = \pm x$  parabolic

$-x < y < x$  elliptic

hyperbolic  $y > x$ ,  $y < -x$

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classify P.D.E

$$x u_{xx} + 4 u_{xy} = 0$$

$$h = 0, \quad a = x \quad b = 4$$

$$h^2 - ab = -4x$$

$x = 0$  parabol

$x < 0$  hyp

$x > 0$  elliptic





Solution of P.D.E

حل المعادلات التفاضلية الجزئية

① الكلام الب سر

Ex

$$\frac{\partial u}{\partial x} = x + y \quad u(0, y) = e^y$$

الطرق النظام الجزئي x-y  
 $u(x, y) = \frac{x^2}{2} + xy + f(y)$   
الثابت ذلك في المتغير ف

$$u(0, y) = e^y = 0 + 0 + f(y)$$

$$f(y) = e^y$$

$$\therefore u = \frac{x^2}{2} + xy + e^y$$

\* Solve P.D.E  
Find solution of P.D.E

$$u_{xx} = x + y$$

$$u_x = 0 \quad u = 0$$

on  $x + y = 0$



$$\frac{\partial^2 u}{\partial x^2} = x + y$$

خارجي:  $y$  ثابت

$$u_x = \frac{x^2}{2} + yx + f(y)$$

$$y = -x$$

$$u_x = 0$$

$$0 = \frac{y^2}{2} - y^2 + f(y)$$

$$f(y) = \frac{y^2}{2}$$

$$\therefore u_x = \frac{x^2}{2} + xy + \frac{y^2}{2}$$

خارجي:  $y$  ثابت

$$u(x, y) = \frac{x^3}{6} + \frac{x^2 y}{2} + \frac{xy^2}{2} + g(y)$$

$$u = 0 = \frac{-y^3}{6} + \frac{y^3}{2} - \frac{y^3}{2} + g(y)$$

$$y = -x$$

$$g(y) = \frac{y^3}{6}$$

$$\therefore u = \frac{x^3}{6} + \frac{x^2 y}{2} + \frac{xy^2}{2} + \frac{y^3}{6}$$

Find Solution of P.D.E

$$u_{xy} = x + y$$

$$u(0, y) = e^y$$

$$u(x, 0) = \cos x$$

حل

$$u_y = \frac{x^2}{2} + yx + f(y)$$

بالتكامل على y:

$$u(x, y) = \frac{x^2 y}{2} + \frac{y^2 x}{2} + F(y) + g(x)$$

①

$$F(y) = \int f(y) dy \quad \leftarrow$$

$$u(0, y) = e^y = 0 + F(y) + g(0)$$

$$\therefore F(y) = e^y - g(0)$$

بالتعويض

$$u = \frac{x^2 y}{2} + \frac{xy^2}{2} + e^y - g(0) + g(x)$$

② بتعويض الشرط

$$u(x, 0) = \cos x$$

①



$$\cos x = 0 + 1 - g(0) + g(x)$$

$$g(x) = \cos x + g(0) - 1$$

الحدود:

$$u = \frac{x^2 y}{2} + \frac{xy^2}{2} + e^y - g(0) + \cos x + g(0) - 1$$

Find sol. solution of P.D.E

$$u_{xx} = x + y$$

$$u(0, y) = e^y$$

$$u(1, y) = e^y + y/2$$

الحدود:

$$u_x = \frac{x^2}{2} + xy + f(y)$$

الحدود:

$$u(x, y) = \frac{x^3}{6} + \frac{x^2 y}{2} + x f(y) + g(y)$$

$$u(0, y) = e^y = 0 + g(y)$$

$$\therefore g(y) = e^y$$

///

$$u(x, y) = \frac{x^3}{6} + \frac{x^2 y}{2} + x f(y) + c$$

$$u(1, y) = e^y + y/2$$

$$= \frac{1}{6} - \frac{y}{2} + f(y) + e^y$$

$$f(y) = y - \frac{1}{6}$$

$$\therefore u(x, y) = \frac{x^3}{6} + \frac{x^2 y}{2} + x \left[ y - \frac{1}{6} \right] + e^y$$

linear equation

$$\frac{\partial u}{\partial x} + p(x) u = q(x, y)$$

$$\int p(x) dx$$

$$u = e$$

$$u \cdot e = \int u \cdot q(x, y) dx + f(y)$$

(11)



Find Solution of P.D.E

$$\frac{\partial u}{\partial x} + \frac{1}{x} u = x + y$$

Q. 16)

$$u(1, y) = \frac{1}{3} + e^y$$

$$P = \frac{1}{x}$$

$$Q = x + y$$

$$v = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$u \cdot \cancel{x} = \int x \cdot (x + y) \cdot d\cancel{x} + f(y)$$

$$\frac{u}{\cancel{x}} x = \frac{x^3}{3} + \frac{x^2 y}{2} + f(y)$$

$$\therefore u(x, y) = \frac{x^3}{3} + \frac{x^2 y}{2} + \frac{1}{x} f(y)$$

$$u(1, y) = \frac{1}{3} + e^y = \frac{1}{3} + \frac{y}{2} + f(y)$$

$$f(y) = e^y - \frac{y}{2}$$

$$\therefore u(x, y) = \frac{x^3}{3} + \frac{x^2 y}{2} + \frac{1}{x} \left[ e^y - \frac{y}{2} \right]$$

(12)

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Solve P.D.E

$$u_{xx} + \frac{1}{x} u_x = x + y$$

let  $u_x = z$   $u_{xx} = z_x$

$$z_x + \frac{1}{x} z = x + y \quad \text{linear}$$

$$\mu = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$z \cdot x = \int x \cdot (x + y) dx + f(y)$$

$$z \cdot x = \frac{x^3}{3} + \frac{x^2 y}{2} + f(y)$$

$$z = \frac{x^2}{3} + \frac{xy}{2} + \frac{1}{x} f(y)$$

$$\frac{\partial u}{\partial x} = \frac{x^2}{3} + \frac{xy}{2} + \frac{1}{x} f(y)$$

$$u(x, y) = \frac{x^3}{9} + \frac{x^2 y}{4} + f(y) \ln x + g(y)$$

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Find solution of P.D.E

$$u_{xx} = \frac{y}{1+x^2}$$

$$u(0, y) = e^y$$

$$u(1, y) = y \left( \frac{\pi}{4} - \frac{1}{2} \ln 2 \right)$$

Ans

$$u(x, y) = y \tan^{-1} x + f(y)$$

$$u(x, y) = y \left[ \tan^{-1} x \right] + x f(y) + g(y)$$

$$\int \tan^{-1} x \, dx$$

$$u = \tan^{-1} x$$

$$du = \frac{1}{1+x^2} dx$$

Integration by parts

$$dv = dx$$

$$v = x$$

$$\int \tan^{-1} x \, dx = x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \ln(1+x^2)$$

Ans

$$u = y \left[ x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) \right] + x f(y) + g(y)$$

$$u(0, y) = e^y = y \left[ 0 - \frac{1}{2} \ln(1) \right] + 0 + g(y)$$

$$\therefore g(y) = e^y$$

$$\therefore u = y \left[ x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) \right] + x f(y) + e^y$$

$$u(1, y) = y \left[ \frac{\pi}{4} - \frac{1}{2} \ln 2 \right] = y \left[ \frac{\pi}{4} \cdot 1 - \frac{1}{2} \ln(2) \right] + f(y) + e^y$$

$$f(y) = -e^y$$

$$\therefore u(x, y) = y \left[ x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) \right] - x \frac{e^y}{17} + e^y \quad \#$$



Find Solution of P.D.E

$$u_{xy} + \frac{1}{x} u_y = x + y$$

let  $u_y = z$        $u_{xy} = z_x$

$$z_x + \frac{1}{x} z = x + y$$

linear

$$z = \frac{x^2}{3} + \frac{xy}{2} + \frac{1}{x} f(y)$$

$$\frac{\partial u}{\partial y} = \frac{x^2}{3} + \frac{xy}{2} + \frac{1}{x} f(y)$$

Integrate w.r.t y

$$u(x, y) = \frac{x^2 y}{3} + \frac{xy^2}{4} + \frac{1}{x} F(y) + g(x)$$

$$F(y) = \int f(y) dy$$

✓